

# Critical temperature for shape transitions in excited nuclei

V. Martín<sup>1</sup>, J.L. Egido<sup>2,a</sup>, and L.M. Robledo<sup>2</sup>

<sup>1</sup> Análisis Numérico, Facultad de Informática, Universidad Politécnica de Madrid, E-28660 Madrid, Spain

<sup>2</sup> Departamento de Física Teórica, Universidad Autónoma de Madrid, E-28049 Madrid, Spain

Received: 31 October 2002 /

Published online: 17 February 2004 – © Società Italiana di Fisica / Springer-Verlag 2004

**Abstract.** The behavior of shell effects with temperature is studied within the framework of the finite-temperature Hartree-Fock-Bogoliubov theory with the Gogny force. Thermal shape fluctuations in the quadrupole degree of freedom are taken into account in the frame of the Landau theory. Numerical results for the superfluid-to-normal and deformed-to-spherical phase transitions are presented for the nucleus <sup>164</sup>Er. We find that the critical temperature for the deformed-spherical shape transition is much lowered when the thermal shape fluctuations are considered.

**PACS.** 21.60.Jz Hartree-Fock and random-phase approximations – 21.10.Ky Electromagnetic moments – 21.10.Ma Level density

One of the most striking features of a heated nucleus is that physical effects like superfluidity or shape deformations are washed out when  $T$  increases [1]. In terms of the shell model, this can easily be understood since by increasing  $T$  one promotes particles from levels below the Fermi surface to levels above it. In the case of pairing correlations, blocking levels amounts to destroying Cooper pairs. In the case of shape deformation, by depopulating the deformation driving levels (intruders), one gets, on the average, less deformation. Experimental information about nuclear shape changes can be obtained by means of the Giant Dipole Resonance (GDR) built on excited states. Exclusive experiments studying the GDR strength at a given excitation energy (or  $T$ ) of the nucleus have been carried out in [2]. In the finite-temperature mean-field theory these effects show up as a phase transition at critical  $T$ 's in the range 0.5–3.0 MeV. It is clear that, as the nucleus is a finite system, the sharp phase transitions obtained in the mean-field approach will be somewhat washed out when statistical fluctuations around the mean-field solution are considered. The statistical fluctuations can be treated in the spirit of the Landau theory [1].

In a recent letter [3], we have reported on Finite Temperature Hartree-Fock-Bogoliubov (FTHFB) calculations performed with the Gogny force [4] for several nuclei. We found that the pairing correlations collapse at temperatures around  $T \approx 0.5$ –1.0 MeV and that deformed nuclei become spherical around  $T \approx 2.0$ –2.5 MeV. The purpose of the present work is to investigate the effects of the shape fluctuations on several quantities in the nucleus <sup>164</sup>Er with the Gogny force. The solution of the FTHFB equa-

tions provide us with the configuration that minimizes the grand canonical potential. At finite temperatures, fluctuations around this solution are important. For nuclei, the most important are the quadrupole ones. To generate the solutions with different shapes, we solve the grand canonical potential equation with an additional constraint on the quadrupole moment. According to Landau, the probability for a certain value  $q$  of the deformation is characterized by the free energy  $F(q)$  of the system with this average deformation  $q$ :

$$P(q) \propto e^{-F(q)/T}. \quad (1)$$

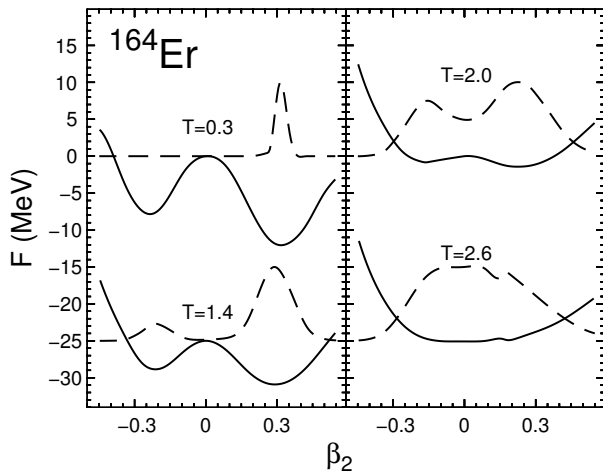
Using classical statistics, the following expression is obtained for the ensemble average of an observable  $\hat{O}$ :

$$\bar{O} = \frac{\int \mathcal{O}(q) \exp(-F(q)/T) dq}{\int \exp(-F(q)/T) dq}, \quad (2)$$

where  $\mathcal{O}(q)$  is the thermal expectation value of the operator  $\hat{O}$  calculated for the system with the deformation  $q$ , and  $dq$  is the volume element in deformation space.

In fig. 1 we present the free energy,  $F(\beta_2)$ , and the quantity  $P(\beta_2) = \exp(-F(\beta_2)/T)$  versus the quadrupole deformation,  $\beta_2$ , at different temperatures.  $P(\beta_2)$  provides the weight of a given shape  $\beta_2$  in the evaluation of thermal average values. At low temperature ( $T = 0.3$  MeV), there is a deep prolate minimum at  $\beta_2 \approx 0.3$  and about 4.5 MeV above an oblate one. The probability distribution  $P(\beta_2)$  is strongly peaked around the prolate minimum. At higher temperatures, the expected disappearance of shell effects becomes clear, in particular the vanishing of the barriers when several minima are available and the development of only one spherical minimum. Further finite-temperature

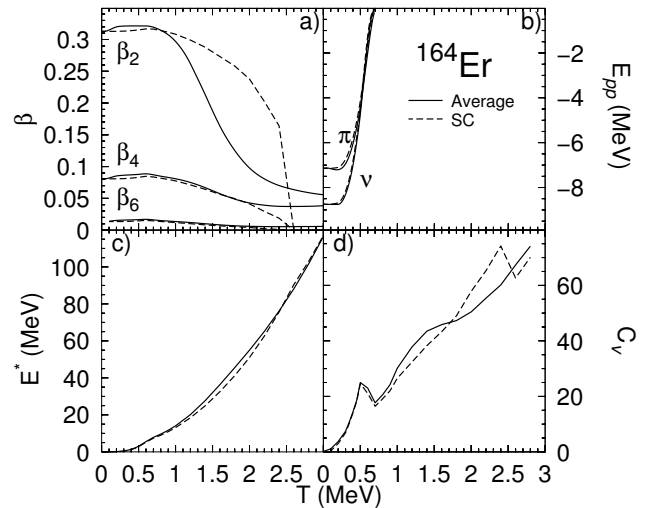
<sup>a</sup> e-mail: j.luis.egido@uam.es



**Fig. 1.** Free-energy curves for  $^{164}\text{Er}$  (continuous lines) at several temperatures as a function of the quadrupole deformation  $\beta_2$ . Free-energy curves are displaced so that the energy difference between two consecutive curves at zero deformation is 25 MeV. The probabilities of eq. (1) (dashed lines) are normalized in such a way that the maximum values are equal to one.

effects like the widening of the free-energy curve and the growing role of fluctuations with increasing temperature appear clearly in this figure.

Figure 2 shows detailed calculations of different quantities as a function of temperature. The dashed lines indicate self-consistent (SC) results, *i.e.* in the absolute minimum of the free-energy curve. The solid lines are used for averaged calculations, see eq. (2). In panel a) the  $\beta_2$ ,  $\beta_4$  and  $\beta_6$  deformation parameters are displayed. Let us first discuss the deformation parameter  $\beta_2$ . For temperatures  $0 < T < 1.0$  MeV, both calculations behave similarly as one would expect for a nucleus with a well-pronounced minimum. For temperatures  $1.0 < T < 2.0$  MeV, the self-consistent  $\beta_2$ -values decrease rather smoothly while the averaged ones experience a strong reduction. For  $T > 2.0$  MeV, the self-consistent values decrease very steeply and collapse, finally, to zero deformation at  $T = 2.6$  MeV. The averaged values, on the contrary, change tendency decreasing very smoothly in such a way that an almost constant value of  $\beta_2$  is obtained. The behavior of  $\beta_4$  and  $\beta_6$  is similar to the one of  $\beta_2$ , though not that spectacular. Looking at the probability distribution in fig. 1, one can easily understand the differences between both calculations. In fig. 2b) the pairing energies are displayed. Up to  $T \approx 0.2$  MeV, the pairing energies are rather constant but for higher  $T$  values they decrease in absolute value very fast up to  $T = 0.7$  MeV where they vanish. Thermal shape fluctuations, as expected in the low-temperature regime, have little effect on the pairing correlations. Pairing fluctuations which would be more relevant are not considered in this work. In panel c) the internal excitation energy,  $E^*$ , is represented. In the low-temperature regime, we can see the pairing collapse which is visible as a change in the slope of  $E^*$ . Above the pairing collapse, in the high-temperature regime, a fairly quadratic behavior is observed in the excitation energy which is slightly altered when the transition



**Fig. 2.** The deformation parameters  $\beta_2$ ,  $\beta_4$  and  $\beta_6$  (a), the pairing energies (b), the excitation energy (c) and the specific heat (d) as a function of temperature.

to the spherical phase takes place at a temperature in the range  $T \sim 1.5$ – $2.5$  MeV. There, a weaker change in the slope can be appreciated, not visible in the scale of the plot. The changes are more abrupt in the SC results than in the average values, again as expected. To analyze in more detail the phase transitions, we show in panel d) the specific heat  $C_v(T) = \partial E^*/\partial T$ . The appearance of peaks in this quantity is considered as a signature for phase transitions. In the self-consistent results we find two peaks, one at  $T \sim 0.7$  MeV, associated with the superfluid-to-normal transition, and one at  $T \sim 2.5$  MeV, associated with the deformed-to-spherical transition. If the shape fluctuations are considered, the low-temperature peak remains largely unaffected. The peak at  $T \sim 2.5$  MeV in the SC calculation, however, is shifted to lower temperatures in the averaged one, where it appears as a broad peak centered at  $T \sim 1.3$  MeV. The broadness of the peak could indicate that the transition takes place in a gradual manner. The reason for this lowering in the critical temperature can be seen in fig. 1. The nucleus  $^{164}\text{Er}$  presents, along with the prolate deformed ground state, a pronounced minimum at oblate deformation, the average of prolate and oblate shapes at finite  $T$  produces spherical shapes at lower  $T$ 's than in the self-consistent solution. This shift has also been found in other nuclei [5].

This work has been supported in part by DGI, Ministerio de Ciencia y Tecnología, Spain, under Project BFM2001-0184.

## References

1. J.L. Egido, P. Ring, J. Phys. G **19**, 1 (1993).
2. A. Atac *et al.*, Phys. Lett. B **252**, 545 (1990).
3. J.L. Egido, L.M. Robledo, V. Martin, Phys. Rev. Lett. **85**, 26 (2000).
4. J.F. Berger, M. Girod, D. Gogny, Comput. Phys. Commun. **63**, 365 (1991).
5. V. Martin, J.L. Egido, L.M. Robledo, Phys. Rev. C **68**, 034327 (2003).